中国农业大学学报　1997，2(4)：59～65
Journal of China Agricultural University

Computerize Real-time Irrigation Scheduling Under Limited Water Supply

Vonlaya Viriyasenakul① Huang Xingfa② Wang Wei Zeng Dechao
(College of Machinery Engineering, CAU)

Abstract The problem of real-time irrigation scheduling under limited water supply is considered. The goal is to develop an irrigation scheduling which maximizes crop yields and response to current season changes in weather and other variables. The irrigation decisions is solved by two-stage process. In the design stage, irrigation scheduling is planned for the entire season based on estimated real-time values of the inputs to the system. To obtain optimal allocations of water for each growth stage, the dated water production function is maximized by dynamic programming. In the operation stage, the water allocated to each growth stage is revised after updating the status of the system with real time data within the stage in a sequential order. Thus, an optimal irrigation water allocation is made. The procedure is illustrated by application to winter wheat under various levels of seasonal water supply initial soil moisture.

Key words real-time irrigation scheduling; dynamic programming; dated water production function

A number of irrigation scheduling techniques and programs have been developed which utilize real-time estimates of ET. Optimization of irrigation management strategies is often determined with simulation model using histrorical meteorological data sets for estimating daily ET. Advances in ET included a number of papers described the use of ET data for irrigation scheduling. Most of these procedures used a water budgeting technique for predicting irrigation time and amount for meeting crop water requirements. One of the major inputs to a water budget is the estimation of daily ET for an individual crop. Most programs require real-time input of climatic data for estimating the actual ET. They all require some techniques of forecasting ET requirements to predict the time and amount of the next irrigation. A common procedure is to first calculate a reference ET based on measured meteorological data. Actual crop ET is then determined by multiplying the reference ET and a crop coefficient, which is a function of the particular crop and its stage of growth and the availability of soil water. Many researchers have measured soil water content to determine crop water use and developed em-

---

Manuscript received: 1996-12-10

①Doctor degree student from Thailand

②China Agricultural University, Box 146, Qinghua Donglu, Beijing, China (Postcode: 100083) pirical equations to calculate crop ET as a function of climatic data.

1 Literature Review

T. A. Howell and E. A. Hiller[1] developed empirical yield relationships for grain sorghum based on ET in order to increase water use efficiency. They determined the response of grain sorghum to water deficits during growth stages. They found that the timing of the deficit is more critical to yield effects than the magnitude of the deficit. T. A. Howell, et al. [2] used the empirical yield relationships to optimize the yield of grain sorghum under limited high frequency irrigation. They used dynamic programing to maximize yield subject to irrigation water availability constraints. They found that the irrigation amount could be significantly reached without a large decrease in yield if the irrigation applications were timed optimally.

A number of models for scheduling irrigations under limited water supplies have been developed in recent years. Many models[3,4] determine the optimal irrigation strategies using stochastic or probabilistic models of weather variables. But in any season,the current weather variables are significantly different from their probabilistic or stochastic estimates. Use of real-time or current weather data to guide irrigation has been attempted mainly by adopting a “trigger level” concept of available soil water. The model assume that the best irrigation strategy is known and do not consider changes in crop response to water stress at different growth stages. There have also been some limited attempts to adapt crop growth simulation models to real time irrigation scheduling. But none of these studies considered the case of real-time irrigation scheduling under a deficient water supply. Deficit irrigation scheduling requires reliable predictions of the effects of irrigation on crop yield. Because of uncertainty of the weather and water supplies, one cannot satisfactorily predict these effects. This study shows a method to guide irrigation system operators in implementing irrigation strategy of a crop growth stage under a deficit water supply in real time. The model for optimal irrigation scheduling previously developed by N. H. Rao, et al. [5,6] is used to formulated and solve the real time irrigation scheduling problem.

2 Problem Formulation

A prerequisite to a flexible or adaptive irrigation operations policy is the definition of the present state of the system. This requires that periodically the actual values of the weather and water supply variables are available to derive the current state of the soil-plant-water supply system. The modus operand in such a case would be cyclic in nature and comprise two stages. The first stage establishes the current state of the system periodically based on actual information up to that period. The second stage determines the optimal course of action in future periods based on predicted future states of the system. Adaptive management of irrigation operations implies treating the problem as “feed-forward” control process. Thus, in the operation phase, the interest is mainly in the irrigation decision to be made in the current decision interval. When it is time to make the next decision, the situation may have changed, necessitating a revision of data and the solution of the problem all over again. However, to compute the decision in the current interval it is necessary to solve the optimization problem completely even though the decisions in the future may not be of specific interest. Thus, even when the operational decision is required only for the next period of interest, the entire planning horizon must be considered to derive it. This is basic to the procedure of real-time adaptive irrigation management under limited water supply.

The problem of adaptive management of irrigation is formulated in two phases: the design or plan phase and the operation phase.

In the design phase, a feasible irrigation plan for the future season is developed based on the probable values of the inputs to the system, namely: rainfall, evaporation, water supplies, etc. for the entire season.

In the operation phase, the future operations to the end of the season are updated at the end of every decision growth stage based on: 1) the state of the system as determined by the actual values of input variables of weather and water supplies up to the current decision growth stage, and 2) the corresponding probable values of the input variables for the subsequent growth stages.

3 Mathematical Formulation

The mathematical formulation used in this study is derived from N. H. Rao, et al. [7] Optimization model is set up based on a time-related water production function. A weekly soil water balance model[8] and a heuristic assumption that water stress in the early weeks of a crop growth stage leads to suboptimal yields. The model used potential ET and rainfall at specific growth stages as the input data. The output from the model was a sequence of the irrigation decisions of each growth stage which maximized the crop yield.

The crop's growing season is divided into N periods which coincide with its physiological growth stages. Let Q_0/mm be the total depth of water available for irrigation at the beginning of the growing season and W_0/mm the initial available soil water.

To solve this problem, the dated water-production function model[7] used is

$$Y/Y_m = \prod \left[1 - K_i(1 - e_{ai}/e_{pi})\right] \tag{1}$$

where Y and Y_m is actual and maximum (when $e_a = e_p$) crop yield respectively; K_i is the yield sensitivity factor, e_{ai} and e_{pi} are, respectively, the actual and potential evapotranspiration in the i growth stage.

The value of e_{ai} is estimated from a soil-water balance model using the procedure given by [9]. The value of e_{pi} is estimated from Penman method described by [10]. The expected rainfall in this period r_i and irrigation applied U_i are assumed to be added to the soil reservoir at the beginning of the growth stage.

If X_i is the water allocated to the i stage:

$$X_i=U_i, i=1,2,\cdots,N$$

To obtain optimal allocations of water to growth stages, equation (1) is maximized by dynamic programming using the backward recursion procedure and the recursive equation:

$$f_i(Q,W)=\max[1-K_i(1-e_{ai}/e_{pi})]f_{i+1}(Q-X_i,W_{i+1}) \tag{2}$$

$$0\leqslant X_i\leqslant Q$$

$$0\leqslant Q\leqslant Q_0$$

and

$$i=N-1,N-2,\cdots,1$$

$$f_N(Q,W_N)=1-K_N(1-e_{aN}/e_{pN}) \tag{3}$$

$$0\leqslant X_N\leqslant Q$$

$$0\leqslant Q\leqslant Q_0$$

In equations (2) and (3), e_{ai} and e_{aN} are respectively a function of X_i, Q and W are the two state variables, namely: available water supply over the remaining season and the avaiable soil water, respectively, at the beginning of each stage.

4 Relevant Real-time Data

In view of the above, real time data of only the following variables are used in each growth stage in the model:

1) actual rainfall, R_{ai}, in the growth stages;

2) updated estimates of seasonal water supplies, Q_{ei}, at the end of the growth stages;

3) irrigation decisions, D_{ai}, actually implemented in the growth stages;

4) actual potential evapotranspiration, e_{api}, in the growth stages, if this is significantly different from the design value e_{pi}.

5 Procedure for Adaptive Management in Real-time

The procedure for adaptive management of irrigation operations begins in the first growth stage of the crop season ($i=N_b$) and continues at the following to the end of the season ($i=N_e$).

1) At the beginning of the first growth stage of the season ($i=N_b$) the design stage irrigation decisions ($X_i, i=N_b, N_b+1,\cdots,N_e$) are determined for the entire season based on the values of rainfall at probability of exceedance ($R_i, i=N_b, N_b+1,\cdots,N_e$) and average potential evapotranspiration ($e_{pi}, i=N_b, N_b+1,\cdots,N_e$) by using the optimization model of N. H. Rao, et al.[5] for specified seasonal water supply Q. X_i is the stipulated irrigation decision for the i growth stage.

2) At the end of the i growth stage, the values of actual rainfall R_{ai}, irrigation decision implemented D_{ai} and the updated estimate of seasonal supplies Q_{ei}, are obtained.

3)If any one of the variables R_{ai}, e_{pi} or Q_{ei} is not significantly different from its corresponding value used in determining the design irrigation programme,this programme continues to remain effective and the operational phase decisions will be the same as those of the design phase for the $(i+1)$ growth stage also.

4)If the value of any one of the variables R_{ai}, e_{pi} or Q_{ei} is significantly different from its corresponding value used in step 1),the design irrigation programmes for the subsequent growth stages need to be modified.

5)This is done by changing the values of the input variables of the growth stage i,namely:R_t, e_{pi} and Q to R_{ai}, e_{api} and Q_{ei},respectively. The corresponding values of these variables for the following growth stages$(i+1, i+2, \cdots, N_e)$ remain the same as those used earlier in the design phase. Further,the irrigation water allocation in this growth stage is set to the actual irrigation applied $(X_i=D_{ai})$. The irrigation decisions are obtained for all subsequent growth stages for these values for the i growth stage once again by running the irrigation scheduling model of N. H. Rao, et al.[5,7]. X_{i+1} obtained from the model is the stipulated irrigation decision for the $(i+1)$ growth stage.

6)i is now set to $(i+1)$ and the procedure(step 2)～5) above)is extended to all the growth stages until the end of the crop season.

7)According to the field condition of this study,there are some limitations of the operation phase for water allocated to i stage,X_i:

X_i can not be more than Q_0, that is $X_i \leqslant Q_0$;

X_i can not be less than 40 mm or more than 80 mm,that is $40 \leqslant X_i \leqslant 80$.

6 Application

The procedures described above were applied to winter wheat in this study. The crop was divided into 6 stages(Table 1). The values of K_c was taken directly from Gong Yuanshi, et al.[10]. The values of K_i taken directly from Chen Yaxin and Kang Shaozhong[11] are assumed to hold for the situation of this study. Values of rainfall,e_0 and e_p were calculated based on the real-time meteorological data during crop growth stages, which were estimated from Penman method. e_a were estimated from a soil-water balance model.

Tab. 1 Growth Stages,K_c,K_t,R_i,e_0 and e_p of Winter Wheat

Stages	No. of Days	K_c	K_t	R_i/mm	e_0/mm	e_p/mm
Sowing—Tillering	27	0.76	0.113 2	2.8	2.3	41.35
Tillering—Greening	122	0.33	0.024 1	2.2	1.5	52.47
Greening—Jointing	40	0.86	0.061 0	15.0	3.6	100.97
Jointing—Heading	19	1.09	0.221 4	2.3	4.2	86.55
Heading—Filling	29	1.22	0.394 4	3.6	4.2	170.19
Filling—Maturity	6	0.41	0.115 0	0.0	5.1	12.55

For the above conditions, the optimal irrigation water allocations X_i to the 6 growth stages of winter wheat, for different levels of seasonal supply Q_0 and initial soil moisture W_0 were presented in Table 2.

Tab. 2 Optimal Irrigation Water Allocation X_i to Growth Stages i for Winter Wheat

mm

Q_0	W_0	X_1	X_2	X_3	X_4	X_5	X_6	Y/Y_m
200	40	0	0	0	60	60	70	0.87
	45	0	0	0	60	60	70	0.87
	50	0	0	0	60	60	70	0.87
250	40	0	0	70	80	0	70	0.87
	45	0	0	0	70	60	70	0.87
	50	0	0	0	70	60	70	0.87
300	40	0	0	60	70	60	70	0.89
	45	0	0	60	70	60	70	0.89
	50	0	0	60	70	60	70	0.89
350	40	80	0	60	70	60	70	0.98
	45	80	0	60	70	60	70	0.98
	50	80	0	60	70	60	70	0.98
400	40	60	70	60	70	60	70	1.00
	45	60	70	60	70	60	70	1.00
	50	60	70	60	70	60	70	1.00

7 Discussion

The expected relative yield and optimal irrigation decision were shown in Table 2. This table showed generally the importance of irrigation timing significantly associated with irrigation quantity. Winter wheat had a duration of about 8 months. The critical period for moisture was from jointing stage to maturity. The decisions were affected by the initial soil water states. When available supplies were highly deficient, irrigation decisions were governed more by crop and irrigation system-related factors than by the soil water availability. However, when supplies approach adequate levels, irrigation frequencies rose and W_0 influenced the timing of irrigations significantly.

8 Conclusion

Computerize real-time irrigations decision-making is visualized as a sequential two-stage process: the design and the operation stages. In the design stage, an irrigation plan for each growth stage of the entire season is developed and updating them for the remaining season, as real time information on the weather, water supply in the operation stage. The mathematical formulation presented above is applied to winter wheat under various levels of seasonal water supply and initial soil moisture. It is particularly suited for such adaptive management.

References

1 Howell T A, Hiller E A. Optimization of water use efficiency under high frequency irrigation: I. ET and yield relationship. Trans of the ASAE, 1975, 18(5): 873～878

2 Howell T A, Hiller E A, Reddell D L. Optimization of water use efficiency under high frequency irrigation: Ⅱ. System simulation and dynamic programing. Trans of the ASAE, 1975, 18(5): 879～887

3 Bras R L, Cordova J L. Intraseasonal water allocation in deficit irrigation. Water Resource Res, 1981, 17: 866～874

4 Day A D, Intalap S. Some effects of soil moisture stress on the growth of wheat. Agron J Jan-Feb, 1970, 62: 27～29

5 Rao N H, Sarma P B S, Chander S. Irrigation scheduling under a limited water supply. Agri Water Manag, 1988, 15: 165～175

6 Rao N H, Sarma P B S, Chander S. Real-time adaptive irrigation scheduling under a limited water supply. Agri Water Manag, 1992, 20: 267～279

7 Rao N H, Sarma P B S, Chander S. A simple dated water production function for use in irrigation agriculture. Agri Water Manag, 1988, 13: 25～32

8 Rao N H. Field test of s simple soil-water balance mode for irrigated areas. J of Hydrology, 1987, 91: 179～186

9 Doorenbos J, Kassam A H. Yield response to water. FAO Irrig and Drain Pap, 1979, 33: 193

10 Gong Yuanshi, Lu Jinwen, Huwe B, et al. Estimation of irrigation water requirements for main crop in North China Plain. Acta Agri Univ Pekinensis, 1993, 19(Sup): 82～91

11 Chen Yaxin, Kang Shaozhong. Theory of Irrigation Scheduling With Limited Water Supply. Beijing: Hydro-Electric Power Pub, 1995. 171